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Vulnerability to Multidimensional Poverty in Algeria and Tunisia: A Counting Approach with Bayesian Network Classifiers

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Abstract - This paper assesses vulnerability to multidimensional poverty in Algeria (2013 and 2019) and Tunisia (2012 and 2018). Using Gallardo's (2022) vulnerability-by-mean-risk framework with Alkire and Foster's (2019) M-*gamma* measures, it models the joint probability of poverty and deprivations through multidimensional Bayesian network classifiers. The study fills the evidence gap in the MENA region and extends Gallardo's approach by examining the dimensional vulnerability among the vulnerable individuals and the overlap with poverty. Results show that Tunisia exhibits higher vulnerability than Algeria despite similar multidimensional poverty. Vulnerability declined overall, and moderate vulnerability dominates in both countries but trends diverged, Algeria shifted towards moderate vulnerability (2013-2019), while Tunisia moved towards severe vulnerability (2012-2018). Chronic poverty is more prevalent in Tunisia than in Algeria, and health and education dimensions are key in distinguishing severe from moderate vulnerability in both countries. These findings highlight contrasting trajectories of vulnerability components in both countries.

Classification JEL
I31, I32, D63, D81

Mots-clés
- Vulnerability
- Multidimensional poverty
- Bayesian network classifier
- Downside mean semi-deviation
- Algeria
- Tunisia

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INTRODUCTION

Like many other countries around the world, MENA countries have adopted the UN agenda 2030 and the Sustainable Development Goals (SDGs). At the heart of the SDGs lies the principle that "no one should be left behind". Yet, the first goal – ending poverty in all its forms everywhere – remains one of the most challenges in the MENA region, given its fragile context and recurrent crises. Recent economic shocks have underscored the need to address not only current poverty but also the vulnerability of those at risk of falling into poverty. A better understanding of vulnerability can help design more effective and sustainable poverty reduction policies.

The inclusion of vulnerability in poverty analysis dates back to the 2000s, following the World Bank's pioneering study on social risk and management (2001). Various approaches have since been proposed, but few have been widely applied. This is largely because vulnerability is a forward-looking concept, and most measures require long panel data, while many countries have only cross-sectional data. This constraint narrows the range of feasible concepts and measures. Moreover, although poverty is widely recognized as a multidimensional phenomenon, vulnerability studies remain dominated by the monetary approach. In reality, vulnerability should reflect multiple dimensions of well-being. However, empirical study on vulnerability to multidimensional poverty is still limited. To date, only a handful of studies exists such as Calvo (2008), Abraham and Kavi (2008), Feeny and McDonald (2016), and the extended cross-dimensional poverty line introduced by OPHI (2018) using the MPI (*Multidimensional Poverty Index from the UNDP*) as a benchmark. Further contributions include Gallardo (2020, 2022) on Latin America and Chile. In the MENA context, research is scarce: apart from Lyons et al. (2021) on Syrian refugees in Lebanon, based on Feeny and McDonald's approach, there are no systematic studies assessing vulnerability to poverty.

This paper aims to fill this gap. Building on Berenger's (2023) analysis of multi-dimensional poverty in Algeria, Iraq and Tunisia, it applies Gallardo's (2022) methodology to assess vulnerability to multidimensional poverty in Algeria and Tunisia. It also explores the relationship between multidimensional poverty and vulnerability: despite similar current levels of multidimensional poverty, do these countries face the same risks of future poverty? Vulnerability is estimated using the downside mean semi-deviation approach (Gallardo, 2013) and the risk of future multidimensional poverty is modeled using Gallardo's (2022) approach which implements multidimensional Bayesian network classifiers. To date, this is only the second application of Bayesian networks in welfare and poverty analysis[1].

The paper is structured as follows. Section 1 reviews the literature on vulnerability to poverty. Section 2 outlines the three-step methodological strategy: (i) multidimensional poverty measurement based on the M-*gamma* family (Alkire and Foster, 2019), Bayesian networks to estimate conditional probabilities, and (iii) the mean-risk approach (Gallardo, 2013) to assess vulnerability. Section 3 presents the results based on UNICEF-MICS data for Algeria and Tunisia. Section 4 concludes with key findings.

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[1] Ceriani and Gigliarano (2020) used Bayesian networks to model dependence structure among different well-being dimensions in selected Western and Eastern European countries.

1. CONCEPTUALIZATIONS AND ASSESSMENTS OF VULNERABILITY TO POVERTY

Poverty and vulnerability to poverty are related but different concepts, both serving as indicators of well-being. Poverty is an ex-post measure, capturing the observed shortfall of household well-being below the poverty line at a given time. It is static and does not account for transitions in and out of poverty. In contrast, vulnerability is an ex-ante concept that reflects the risk of falling into poor in the future due to exposure to shocks and other risks (Calvo and Dercon, 2013). It combines elements of poverty and risk (Chaudhuri et al., 2002). Measuring vulnerability is complex because of uncertainty about future risks.

The literature offers numerous definitions and approaches to vulnerability, but no consensus exists. Key surveys by Hoddinott and Quisumbing (2003, 2008), Ligon and Schechter (2003), Calvo and Dercon (2013), Klasen and Povel (2013), and Gallardo (2018) categorize these approaches into three main types: vulnerability as expected utility (VEU) by Ligon and Schechter (2003), vulnerability as uninsured exposure to risk (VER) by Tesliuc and Lindert (2002), and vulnerability as expected poverty (VEP) by Chaudhuri et al. (2002). Gallardo (2018) added a fourth category: vulnerability by mean risk (VMR). Each category includes multiple approaches, but we focus on the main ones.

VEU measures the difference between a certainty-equivalent level of well-being and the household's expected utility, but it relies on a specific utility function and treats risk symmetrically (Klasen and Povel, 2013; Gallardo, 2018). VER examines welfare loss from shocks due to inadequate risk management[2].

VEP assesses the likelihood that a shock pushes a household below the poverty line; it is widely used, especially in developing countries, due to the availability of cross-sectional data. However, VEP has several limitations: it assumes that past distributions of well-being can predict the future, applies this assumption uniformly across households, and relies on a predefined probability distribution. Gallardo (2018) also criticizes VEP for ignoring risk sensitivity and the depth of expected poverty, focusing solely on the probability of falling below the poverty line.

VMR incorporates the mean deviation approach developed by Chiwaula et al. (2011) as well as the downside mean semi-deviation proposed by Gallardo (2013). These two approaches identify vulnerable people based on a preference ordering of welfare outcomes, considering both the expected mean and a risk parameter: variance in the first approach and downside semi-deviation in the second. Unlike symmetric risk measures, the downside mean semi-deviation acknowledges that poverty risk is asymmetric, since households are more concerned with declines below expected well-being. This combines expected poverty with the downside risk. Individual vulnerability levels can then be aggregated using standard FGT indexes. Initially applied to monetary poverty (Gallardo, 2013), this approach has recently been extended to multidimensional poverty (Gallardo, 2020 and 2022).

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[2] Recent extensions incorporate asymmetric conception of risk, either from lack of insurance against falling below the poverty line (Cafiero and Vakis, 2006) or from downside risk (Dutta et al. 2011 ; Povel, 2010, 2015).

In summary, these approaches model and predict well-being and poverty risk, but differ in their concepts of well-being and in their risk modeling. Most rely on the expected mean and variance of household consumption relative to a benchmark. VEP and VMR can be evaluated with cross-sectional data, whereas VER and VEU require panel data, which limits research. Nevertheless, recent econometric advances have enabled vulnerability estimation with cross-sectional data. Chaudhuri et al. (2002, 2003) proposed a method to estimate expected mean and variance of (log) consumption using FGLS with single cross-section data.

Despite the recognition of the multidimensional nature of poverty, most empirical studies still rely on income or consumption expenditure as proxies. Only recently have a few studies explored vulnerability to multidimensional poverty, mainly applying the VEP approach to households' deprivation scores (Alkire and Foster, 2011; Chaudhuri, 2003). Applications include Feeny and McDonald (2016) in Melanesia, Azeem et al. (2018) in Pakistan, Tigre (2019) in Ethiopia, Gebrekidan et al. (2020) in Ethiopia, Liu et al. (2021) in rural China, Lyons et al. (2021) on Syrian refugees in Lebanon; and Hernandez and Zuluaga (2022) in Colombia. However, this approach reduces multidimensional deprivation to a single score, preventing analysis of vulnerability by dimension and limiting results to incidence, while ignoring severity.

To address some of these issues, Pham et al. (2021) employed Chiwaula et al.'s measure (2011) to examine vulnerability to poverty in Vietman, applying a fuzzy set approach across income and six non-monetary dimensions with panel data. Similarly, Gallardo (2020) applied the mean-risk approach (Gallardo, 2013) in Chile, estimating the probability of being non-poor for each MPI indicator through a multilevel Probit model. This strategy integrates the Alkire and Foster method with dimensional vulnerability and multidimensional poverty thresholds, but it does not fully address multidimensionality.

Gallardo's (2022) study offers a promising alternative. To preserve multidimensionality in vulnerability estimation, Gallardo applied a multidimensional Bayesian network classifier to estimate conditional probabilities of being multidimensional poor, combined with the VMR approach using downside semi-deviation as the risk parameter. The method produces individual-level vulnerability estimates, that can be aggregated into Foster-Greer-Thorbecke (FGT) vulnerability measures, and also allows decomposition by dimensions.

The originality of our paper lies in addressing the lack of studies on the MENA region by applying Gallardo's methodology to two countries with similar levels of multidimensional poverty. It further employs the recent M-*gamma* extension of the Alkire-Foster MPI to assess vulnerability and, unlike Gallardo, examines dimensional vulnerability by combining the headcount of multidimensionally vulnerable individuals with a dimensional breakdown, following the Alkire-Foster approach.

2. METHODOLOGICAL STRATEGY

This section outlines the three-step approach developed by Gallardo (2022). First, multidimensional poverty is estimated. Next, the uncertainty in conditional probabilities of being multidimensional poor and deprived across well-being dimension is modeled using Bayesian network classifiers. Finally, vulnerability is measured using the mean downside semi-deviation as developed by Gallardo (2013, 2022).

2.1. Multidimensional poverty assessment

We use the M-*gamma* family of multidimensional poverty measures by Alkire and Foster (2019) which we use as a benchmark for assessing vulnerability. Alkire and Foster's (2011) counting-based approach uses binary variables and a dual cut-off to identify the multidimensionally poor.

For a population of n individuals $(i = 1,..,n)$, and m well-being indicators $(j = 1,..,m)$, with weights (w_j) assigned to each indicator, two cut-offs are used to identify the multidimensionally poor: dimension-specific poverty lines (z_j) and a cross dimensional cut-off (k). Individual deprivations in each dimension are compared to z_j, and then summed into a weighted deprivation count (c_i). If $c_i \geq k$, the individual is identified as multidimensionally poor. For example, the UNDP's global MPI sets k at one-third of weighted dimensions.

Aggregate poverty levels are computed using the M-*gamma* class of poverty measures:

$$M_0^\gamma = \frac{1}{n}\sum_{i=1}^{n} c_i^\gamma(k) \quad for\, \gamma \geq 0 \tag{1}$$

where $c_i^\gamma(k) = c_i^\gamma$ if $c_i \geq k$, 0 otherwise.

For $\gamma = 0$, H denotes the multidimensional headcount ratio. For $\gamma = 1$, $M_0^1 = H \times A$, the Multidimensional Poverty Index (MPI), with A the average deprivation of the poor (intensity of poverty):

$$M_0^1 = H \times A \tag{2}$$

with $A = \frac{1}{q}\sum_{i=1}^{q} c_i\,(k)$ with q the number of poor individuals.

M_0^1 is decomposable by population subgroup and dimension, highlighting the contribution of each indicator to overall poverty and the deprivation profile of the poor.

For $\gamma = 2$, M_0^2 generalizes the squared poverty gap $(FGT_{\alpha=2})$ to multidimensional settings, capturing inequality among the poor[3].

2.2. Modeling uncertainty using Bayesian network classifiers

Ex-post identification of multidimensionally poor individuals can be used to construct an ex-ante poverty measure by treating multidimensional poverty status and its various indicators as random variables. These binary variables (being poor/not poor, deprived/not deprived) follow a Bernoulli distribution. Since household characteristics affect deprivations, the aim is to estimate a joint probability distribution predicting both the risk of multidimensional poverty and deprivation in each dimension.

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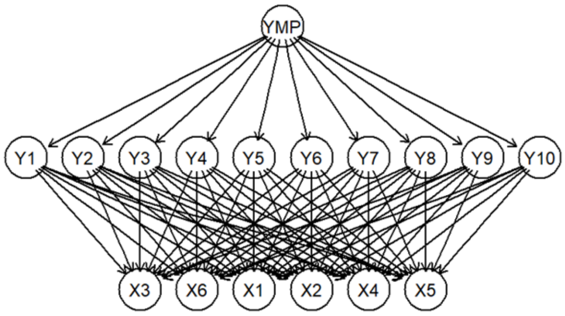
[3] M_0^2 can be decomposed into the three 'I's of poverty (Jenkins and Lambert, 1997): incidence, intensity and inequality of multidimensional poverty:

$$M_0^2 = H \times A^2 \times \left[1 + 2GE_2(c_p)\right] = M_0^1 \times A \times \left[1 + 2GE_2(c_p)\right]$$

Here, $GE_2(c_p)$ represents half the squared coefficient of variation of deprivation scores among the poor and belongs to the Generalized Entropy class of inequality measures.

Figure 1. MBC to estimate conditional posterior probabilities



Note: Our implementation of MBC includes ten indicators used to measure multidimensional poverty as well as six households features as covariates of deprivation in each indicator. Source: Author's.

This approach preserves the multidimensionality of poverty by capturing interdependencies between deprivations and household characteristics.

Consider n individuals. Each individual i described by an m-random vector: $Y_i = (Y_{i1}, \dots, Y_{im})$ where $Y_{ij} = 1$ if the individual is not deprived and 0 otherwise. Additionally, define a n-dimensional random vector: $Y^{MP} = (Y_1^{M}, \dots Y_i^{MP} \dots, Y_n^{MP})$, where $Y_i^{MP} = 1$ if the individual is not multidimensionally poor and 0 otherwise. The realizations of Y_i^{MP} depend on Y_i, influenced by household and community characteristics: $X_i = (X_{i1}, \dots, X_{iq})$.

The uncertainty associated with multidimensional poverty can then be modeled via the joint probability distribution function:

$$P(Y_i^{MP}, Y_i, X_i) = P\big(y_i^{MP}, y_{i1}, \dots, y_{im};\ x_{i1}, \dots, x_{iq}\big), i = 1, \dots, n \qquad (3)$$

The objective is to simultaneously estimate both the probability of being poor/ non-poor given deprivations, $P(y_i^{MP}|y_i)$, and the probability of being deprived/ non-deprived in each dimension given household characteristics, $P\big(y_{ij}\,|x_i\big)$.

A multidimensional Bayesian network classifier (MBC) is particularly suited for this task[4].

As a Bayesian network classifier, it handles multiple class variables by assigning instances with multiple features to combinations of classes (see Zaragoza et al., 2011).

Variables are divided into class variables Y and feature variables $X = \big(X_1, \dots., X_q\big)$, which may be binary or categorical. The classifier uses the joint probability distribution to compute the posterior conditional distribution of the class variable given observed features, with classification based on maximum a posteriori (MAP) estimation.

[4] To ease the presentation in what follows, we omit the indices related to individuals.

In this framework, the classifier has a two-level structure. At the first level, the q feature variables X_j predict m class variables Y_j. At the second level, these m class variables Y_j form a feature vector that predicts the super-class variable Y^{MP} (see Figure 1).

The MBC provides, for each individual i, the posterior conditional probabilities: p_i for the super-class y_i^{MP} and p_{ij} for y_{ij} for each class in dimension $j = 1, \dots, m$. Detailed derivations are presented in Appendix A1.

2.3. Indicators for measuring vulnerability to multidimensional poverty

We use Gallardo's (2013) *Vulnerability by Mean Risk* (VMR), which applies a mean risk criterion to measure vulnerability.

For each individual i, vulnerability is measured by a risk-adjusted mean of well-being:

$$\tilde{\mu}_i = \mu_i - \lambda\tilde{\sigma}_i \quad (4)$$

where μ_i is expected well-being, $\tilde{\sigma}_i$ the downside mean semi-deviation (capturing only risks below the mean), and $\lambda \in [0,1]$ a risk aversion parameter[5]. This parameter reflects the trade-off between the mean and the risk of losses in well-being. The measure focuses only on deviations of well-being below its expected value, consistent with the idea that individuals seek to maximize μ_i and minimize $\tilde{\sigma}_i$.

With z the poverty line under certainty, an individual is considered vulnerable if $\tilde{\mu}_i \leq z$, distinguishing: severe vulnerability ($\mu_i \leq z$) and moderate vulnerability ($\mu_i > z \wedge \mu_i - \lambda\tilde{\sigma}_i \leq z$).

Standard Foster-Greer-Thorbecke (FGT) measures can then be applied directly to $\tilde{\mu}_i$ to derive vulnerability indices.

In the multidimensional case, the same principle applies using the Alkire and Foster counting approach.

Given the MBC estimates of individual probabilities p_i (non-poverty across all dimensions) and p_{ij} (non-deprivation) in each dimension j, the risk-adjusted parameter is:

$$\tilde{\mu}_i^{rp} = p_i - \lambda\, \tilde{\sigma}_i^{rp} \text{ with } \tilde{\sigma}_i^{rp} = [p_i^2(1-p_i)]^{1/2} \quad (5)$$

An individual is considered vulnerable to multidimensional poverty if $\tilde{\mu}_i^{rp} \leq 0.5$.

Aggregate measures of vulnerability to multidimensional poverty (V_α^{MP}) and by dimension $\left(V_\alpha^{JP}\right)$ are defined analogously to FGT measures ($\alpha = 0,1,2$) corresponding to vulnerability headcount, gap and square gap ratios.

Vulnerability to multidimensional poverty V_α^{MP} can be expressed as follows:

$$V_\alpha^{MP} = \frac{1}{n}\sum_{i=1}^{n} g_i^\alpha\, I_{\tilde{\mu}_i^{rp} \leq z^p} \text{ with } \alpha \geq 0$$

[5] The parameter λ weights the trade-off in $\tilde{\mu}_i$: λ=0 implies risk neutrality, while higher values give more weight to avoiding downside risk.

and $g_i = \left(\frac{z^p - \tilde{\mu}_i^{rp}}{z^p}\right)$ the vulnerability gap of individual i relative to the probability threshold z^p=0.5.

These indices can be decomposed, for both V_α^{MP} and V_α^{JP} $(j = 1, \dots, m)$ into poverty-induced (severe) and risk-induced (moderate) components, denoted by subscripts P and R respectively:

$$V_\alpha^{MP} = V_{\alpha,P}^{MP} + V_{\alpha,R}^{MP} \quad (6)$$
$$V_\alpha^{JP} = V_{\alpha,P}^{JP} + V_{\alpha,R}^{JP} \quad (7)$$

In the empirical section, we examine both the overlap between ex-post multidimensional poverty and ex-ante vulnerability and the dimension-specific profiles of vulnerable groups.

3. RESULTS AND DISCUSSION

In our empirical analysis, we focus on Algeria and Tunisia. Both countries are classified as middle-income countries but they have adopted different economic models. While Tunisia based its development on an export-oriented, labor-intensive model and tourism, Algeria is an oil-producing country. Between 2012 and 2018, Tunisia experienced sluggish economic growth with an annual average GDP per capita growth rate of 1.4 % due to instability. In Algeria, where the annual average GDP per capita growth rate was 0.6%, economic performance was highly dependent on oil price volatility. According to the Human Development Index, Algeria and Tunisia are among the highest in the Arab world. Despite their commitment to SDG 1, monetary measures based on international and national poverty lines remain the primary means of monitoring poverty. However, the most recent estimates of monetary poverty date back to 2011 for Algeria and 2015 for Tunisia. Of the limited studies employing a multidimensional approach to poverty in the MENA region, Abu-Ismail et al. (2015) studied Jordan, Iraq, and Morocco, Bérenger (2017) focused on Egypt and Jordan, Bérenger (2023) covered Algeria, Iraq, and Tunisia, Nasri and Belhadj (2017), as well as Ben Hassine and Sghairi (2021) utilised data from the 2010 Tunisian household budget surveys; and Oznur and Eleftherios (2021) examined selected MENA countries. While multi-dimensional poverty studies remain scarce, research measuring vulnerability to poverty in the region is almost non-existent. To our knowledge, the only study is Lyons et al. (2021) on Syrian refugees in Lebanon.

3.1. Data description

We use data from UNICEF's Multiple Indicators Cluster Survey (MICS) for Algeria (2012/13, 2018/19) and Tunisia (2012, 2018). Table 1 lists the indicators, organized along the same three dimensions as the MPI: education, health, and standard of living. In line with ESCWA's proposals for an Arab MPI (2017 and 2021), the cut-off for deprivation in years of education is defined by secondary school completion, while deprivation in school attendance is based on the duration of compulsory schooling. We include three additional indicators:

- Overcrowding, reflecting rising real estate and housing prices in Arab countries;
- child obesity, alongside undernutrition, as a growing concern in the region;
- early pregnancy or marriage (women under 28), as a major contributor to maternal deaths.

The MPI thus comprises ten indicators $(Y_1, \dots, Y_m)$ grouped into three dimensions using the same weighting structure as the UNDP MPI.

The household characteristics used as variables for the MBC are listed in Table 1.A in the Appendix. Due to in data limitations, six variables were selected; denoted as $(X_1, \dots, X_q)$. To implement the MBC, the feature variables were categorized as shown in Table 1.A.

Multidimensional poverty measures were computed using a poverty threshold of $k = 1/3$. For vulnerability measures, the choice of the risk aversion parameter λ is arbitrary; here, we set $\lambda = 1$ and used a vulnerability threshold of 0.5.

Since vulnerability measures are based on poverty measures constructed with the Alkire and Foster approach, we first present the results of these measures. We then report the results from the MBC and the derived vulnerability measures. Finally, we examine the overlap with multidimensional poverty to identify distinct categories of vulnerable people.

Table 1: List of dimensions and indicators

Dimension	Indicators	Deprivation Cut-off	Relative weight
Education	School attendance	Any school-aged child (6-16) is not attending school or is two years or more behind the right school grade	1/6
	Years of education	No household member aged 17 years or older has completed secondary school	1/6
Health	Nutrition	Any child (0-59 months) is stunted or overweight (weight for height > +2SD)	1/9
	Mortality	Any child from a household who has died	1/9
	Early pregnancy or marriage	A woman less than 28 years old got first pregnancy or marriage before being 18 years old	1/9
Standard of Living	Water	No access to safe drinking water source within 30 minutes one-way distance from the residence	1/15
	Sanitation	Household sanitation facility is not improved or improved but shared.	1/15
	Overcrowding	Household has 2.5 people per sleeping room	1/15
	Floor	Household has rudimentary or cement floor	1/15
	Assets	Household has less than two assets for accessing to information (radio, TV, phone) or less than two livelihood assets (refrigerator, washing machine, air conditioner, water heater, stove) and household has less than two mobility assets (car, bike, motorcycle)	1/15

Source: Author's based on UNICEF-MICS data.

3.2. Multidimensional poverty measures

Table 2 reports the multidimensional poverty estimates of the MPI (M_0^1) and its two components, the incidence (H) and intensity (A), as well as M_0^2 for Algeria and Tunisia over two years. A comparison of the two countries reveals similar poverty levels in 2019 and 2018, respectively. To analyze poverty trends, Table 2.A in the Appendix, provides a breakdown by area of residence. At the national level, both countries experienced a reduction in multidimensional poverty, though with notable differences between and within them. Algeria experienced the fastest reduction in its MPI, decreasing from 0.120 in 2013 to 0.049 in 2019 – a yearly decline of 13.80%. This progress enabled Algeria to catch up with Tunisia, which had an even lower initial level of M_0^1 (0.079).

Table 2. Observed multidimensional poverty using the M-gamma family measures

	H	M_0^1	A	M_0^2
Algeria				
2013	0.259	0.120	0.463	0.058
	(0.007)	*(0.003)*	*(0.002)*	*(0.002)*
2019	0.113	0.049	0.437	0.022
	(0.004)	*(0.002)*	*(0.003)*	*(0.001)*
ARC	-0.130	-0.138	-0.010	-0.148
Tunisia				
2012	0.176	0.079	0.451	0.038
	(0.008)	*(0.004)*	*(0.004)*	*(0.002)*
2018	0.112	0.049	0.432	0.022
	(0.006)	*(0.003)*	*(0.004)*	*(0.001)*
ARC	-0.072	-0.078	-0.007	-0.086

Note: ARC is the average annualized change. Standard errors are reported between brackets. ARC are statistically significant at α=0.01.
Source: Author's calculation based on UNICEF-MICS data.

The decline in poverty was driven by simultaneous decreases in both H and A, with Algeria experiencing a significantly faster reduction (13% and 1% per year, respectively) than Tunisia (7.20% and 0.70% per year, respectively).

In Tunisia, rural poverty declined more rapidly than urban poverty, narrowing the urban-rural divide. By contrast, in Algeria poverty measures (M_0^1, H, M_0^2) decreased more slowly in rural than in urban areas, thus widening the gap.

Turning to M_0^2, which is sensitive to inequality among the poor, Table 2 shows that the decline in poverty was accompanied by a decrease in inequality among the poor in both countries. In Tunisia, the decline was faster in rural than in urban areas. However, in Algeria, this pattern was not observed (Table 2.A.), as the urban poorest benefited more from the decline than the rural poorest.

To examine patterns of vulnerability in Algeria and Tunisia, an ex-ante approach to poverty is required.

3.3. Results from the MBC implementation

As described in section 2.2, we implemented the MBC to obtain the posterior conditional probabilities for each individual i, denoted as p_i for y_i^{MP} and p_{ij} for y_{ij} in dimension $j = 1, \dots, m$. These probabilities were then used to construct measures of vulnerability to multidimensional poverty. To assess the predictive accuracy of the Bayesian network classifier, we employed two measures, following similar studies (Gil-Begue et al., 2020; Zaragoza et al., 2011). The first is overall accuracy, which indicates how well the model predicts the values of $y^{MP}$for multidimensionally poor and non-poor individuals. The second is the average accuracy over the class variables y_j, representing the mean prediction accuracy for each dimension. The results are reported in Table 3. The MBC's overall accuracy ranges from 0.83 to 0.90, indicating good predictive performance.

Table 3. Predictive accuracy of the Bayesian network classifiers using five-fold cross validation

	Algeria 13	Algeria 19	Tunisia 12	Tunisia 18
Accuracy by Dimension				
Sanitation	0.88	0.88	0.88	0.97
Water	0.82	0.93	0.93	0.86
Floor material	0.75	0.72	0.72	0.83
Overcrowding	0.75	0.80	0.80	0.89
Assets	0.73	0.73	0.73	0.78
Nutrition	0.91	0.92	0.92	0.95
Early Pregnancy	0.98	0.97	0.97	0.98
Mortality	0.94	0.96	0.96	0.97
School attendance	0.77	0.88	0.88	0.92
Years of Education	0.82	0.83	0.83	0.77
Average accuracy dimensions	0.84	0.86	0.86	0.89
Overall Accuracy	0.83	0.90	0.90	0.91

Note: In order to assess the predictive accuracy of the Bayesian network classifier, we applied a 5-fold cross-validation procedure. The idea behind this procedure is to randomly split the original data set into k-folds (or subsets). For each fold, a model is trained on the k-1 folds of the dataset and the remaining set is used as a validation test. The procedure is repeated until the k-folds have served as test sets. At each step, the accuracy of the model is recorded and cross validation accuracy is simply the average of the k recorded accuracy.
Source: Author's calculation based on UNICEF-MICS data.

Accuracy by dimension shows that the MBC performs best in predicting outcomes related to early pregnancy, mortality, and nutrition across all periods and countries. However, predictions are less accurate (less than 0.8) for indicators related to floor material and assets throughout the entire period in both countries.

3.4. Measures of vulnerability to multidimensional poverty

The probabilities obtained from implementing the MBC were used to compute the risk-adjusted probabilities of each individual being non-poor $\left(\tilde{\mu}_i^{rp}\right)$ or non-deprived in each indicator $\left(\tilde{\mu}_{ij}^{rp}\right)$, as well as the vulnerability measures V_α^{MP} for $\alpha = 0, 1, 2$. However, these measures were derived solely from the individual vulnerability gaps using the information contained in $\tilde{\mu}_i^{rp}$ and thus did not provide insights into the specific components of vulnerability faced by individuals. Moreover, interpretation can be challenging since the measures are expressed in terms of probabilities[6].

To address these limitations, we propose combining the information about the identification of vulnerable individuals through the headcount ratio, with information on vulnerability in each dimension. This approach parallels that of Alkire and Foster in constructing the MPI. The resulting measures, denoted V_{01}^{MP} and V_{02}^{MP}, are analogous to M_0^1 and M_0^2 in multidimensional poverty[7]. Table 4 presents national-level results for each country, while Table 2.A in the Appendix reports results by area of residence.

In all cases, the vulnerability headcount ratios are significantly higher than the poverty headcount ratios, suggesting that current poverty estimates only provide

[6] For instance, V_1^{MP} represents the proportion of the risk-adjusted probability needed to exceed the vulnerability threshold.

[7] A complete assessment of vulnerability also requires examining dimension-specific vulnerability gaps, which can be captured using multidimensional measures proposed in the literature.

an incomplete picture. Although Algeria and Tunisia have similar levels of multidimensional poverty, the estimates in Table 4 show that Tunisia has higher vulnerability measures than Algeria. Furthermore, the vulnerability to poverty ratios (V_0^{MP}/H) in Table 4 indicate that, for every person living in poverty, there are 1.5 vulnerable individuals in Algeria (2019) and 1.8 in Tunisia (2018).

Table 4. Measures of Vulnerability to Multidimensional Poverty using $\lambda = 1$

Multidimensional Vulnerability based on risk-adjusted mean					
	V_0^{MP}	V_1^{MP}	V_{A1}^{MP}	V_2^{MP}	V_0^{MP}/H
Algeria					
2013	0.451	0.266	0.590	0.201	1.741
	(0.007)	*(0.006)*	*(0.006)*	*(0.005)*	
2019	0.176	0.091	0.518	0.072	1.568
	(0.005)	*(0.004)*	*(0.009)*	*(0.003)*	
ARC	-0.145	-0.163	-0.021	-0.157	
Tunisia					
2012	0.317	0.170	0.537	0.129	1.805
	(0.007)	*(0.005)*	*(0.009)*	*(0.00)4*	
2018	0.210	0.112	0.535	0.087	1.867
	(0.008)	*(0.005)*	*(0.010)*	*(0.005)*	
ARC	-0.067	-0.067	-0.001	-0.064	
Multimensional vulnerability based on dimensional vulnerability					
	V_0^{MP}	V_{01}^{MP}	V_{A01}^{MP}	V_{02}^{MP}	V_{02ineq}^{MP}
Algeria					
2013	0.451	0.226	0.501	0.122	0.037
	(0.007)	*(0.004)*	*((0.002)*	*(0.003)*	
2019	0.176	0.075	0.425	0.035	0.047
	(0.005)	*(0.002)*	*(0.002)*	*(0.001)*	
ARC	-0.145	-0.168	-0.027	-0.188	
Tunisia					
2012	0.317	0.138	0.436	0.067	0.054
	(0.007)	*(0.003)*	*(0.003)*	*(0.002)*	
2018	0.210	0.085	0.404	0.038	0.053
	(0.008)	*(0.004)*	*(0.004)*	*(0.002)*	
ARC	-0.067	-0.078	-0.013	-0.091	

Note: ARC is the average annualized relative change. Standard errors are reported between brackets. ARC are statistically significant at α=0.01.
Source: Author's calculation based on UNICEF-MICS data.

Over time, this ratio has decreased in Algeria, whereas it has increased slightly in Tunisia. In addition to poverty, vulnerability has decreased over time in these two countries, but the reduction in vulnerability has been faster in Algeria than in Tunisia, with annual rates of 14.5% and 6.7% respectively for V_0^{MP} across all vulnerability measures.

Comparing the evolution of vulnerability with that of multidimensional poverty provides valuable insights into the paths that poverty has taken in these two countries. Regardless of the approach adopted to measure vulnerability, it decreased at a faster rate than poverty, at both the national level and urban areas (Table 2.A.) in Algeria. However, Tunisia's decrease in vulnerability is slower than its decrease in poverty, at both the national level and rural areas (Table 2.A.). This suggests that progress in reducing poverty is more fragile in Tunisia than in Algeria. Similar trends are also observed in measures that account for the intensity and inequality among the poor. Additionally, decomposing the vulnerability measure V_{01}^{MP}, analogous to M_0^1 or the MPI in Table 4, provides insight into the intensity of vulnerability V_{A01}^{MP} among the vulnerable population. Notably, while

Algeria exhibited lower vulnerability in 2019 than Tunisia in 2018, vulnerable individuals in Algeria are at risk of deprivation in 42.5 % of the well-being attributes compared to 40.4% in Tunisia.

Table 5. Decomposition into severe and moderate vulnerability

	V_0^{MP}	V_{0P}^{MP}	V_{0R}^{MP}	V_{A01P}^{MP}	V_{A01R}^{MP}	V_{A1P}^{MP}	V_{A1R}^{MP}	V_{0P}^{MP}/V_0^{MP}
Algeria								
2013	0.451	0.184	0.267	0.537	0.476	0.367	0.222	0.408
	(0.007)	*(0.006)*	*(0.004)*	*(0.003)*	*(0.003*	*(0.007)*	*(0.004)*	
2019	0.176	0.065	0.111	0.470	0.399	0.349	0.169	0.371
	(0.005)	*(0.003)*	*(0.003)*	*(0.005)*	*(0.003*	*(0.012)*	*(0.004*	
ARC	- 0.145	- 0.158	- 0.136	- 0.022	- 0.029	- 0.009	- 0.045	
Tunisia								
2012	0.317	0.103	0.214	0.528	0.392	0.298	0.239	0.324
	(0.007)	*(0.005)*	*(0.006*	*(0.007)*	*(0.003*	*(0.012)*	*(0.006)*	
2019	0.210	0.091	0.119	0.452	0.367	0.398	0.137	0.434
	(0.008)	*(0.005)*	*(0.004*	*(0.005)*	*(0.005*	*(0.012)*	*(0.004)*	
ARC	- 0.067	- 0.020	- 0.094	- 0.026	- 0.011	0.049	- 0.089	

Note: ARC is the average annualized relative change. Standard errors are reported between brackets. ARC are statistically significant at α=0.001. Values of ARC for V_{A01P}^{MP} and V_{A01R}^{MP} are easier to interpret by considering the absolute variation which gives outcomes in terms of share of weighted dimensions.
Source: Author's calculation based on UNICEF-MICS data.

We now examine the composition of vulnerability into its risk induced $V_{\alpha,R}^{MP}$ and poverty-induced $V_{\alpha,P}^{MP}$ components. We limit our presentation to the decomposition of the vulnerability headcount ratio V_0^{MP} and the intensity of vulnerability within each vulnerable group as our aim is to focus on trends in these two components.

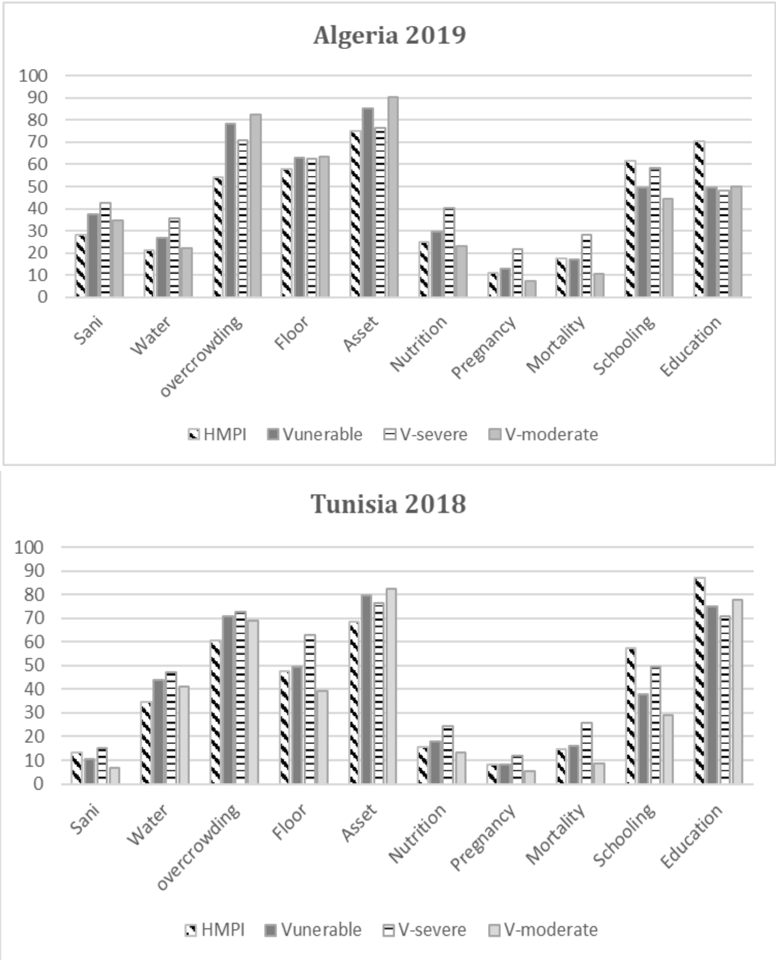
Table 5 provides measures of the intensity of vulnerability: the risk-adjusted probability gap (V_{A1P}^{MP}; V_{A1R}^{MP}) and the proportion of dimensions in which vulnerable individuals face deprivation risk (V_{A01P}^{MP}; V_{A01R}^{MP}).

Table 5 reports $V_{0,P}^{MP}$, representing the percentage of individuals whose vulnerability is due to a low expected level of well-being ("severe" vulnerability in Gallardo (2013)), and $V_{0,R}^{MP}$, representing the percentage of individuals who suffer vulnerability due to the volatility of their well-being ("moderate" vulnerability).

Table 3.A in the appendix also presents the results by area of residence. Table 5 shows that moderate vulnerability is more prevalent than severe vulnerability in both Algeria and Tunisia. Algeria has achieved the largest reductions in both vulnerability components, compared to Tunisia. In Algeria, the decrease in the headcount ratio of severe vulnerability has been faster than that of moderate vulnerability (15.8% and 13.6% resp.), although improvements in the intensity of vulnerability sligthly favored the moderately vulnerable (0.177) over the severely vulnerable (0.067), according to V_{A01R}^{MP} and V_{A01P}^{MP}.

These trends are particularly evident in rural areas (Table 3.A). In urban areas, the trends are more ambiguous despite the significant decrease recorded in the vulnerability headcount ratio. Overall, in Algeria, vulnerability appears to be shifting towards moderate vulnerability, as the contribution of severe vulnerability to overall vulnerability (V_{0P}^{MP}/V_0^{MP}) fell from 40.8% in 2013 to 37.1% in 2019.

Figure 2. Deprivation rates by dimension among the vulnerable

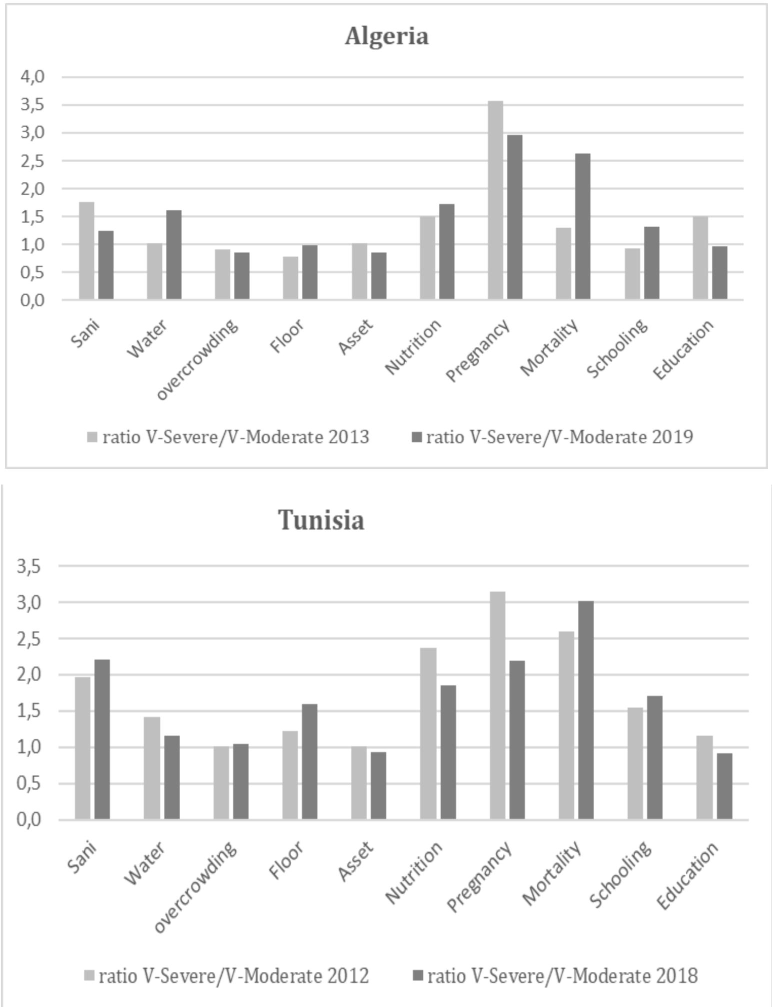


Source: Author's calculation based on UNICEF-MICS data.

On the other hand, opposite trends are observed in Tunisia. The most significant decreases in vulnerability concern the moderate vulnerability group, both in terms of the headcount ratio (V_{0R}^{MP}) and the intensity of the risk of multiple deprivation. These trends are particularly noticeable in rural areas (Table 3.A) for moderate vulnerability. However, regarding the severe vulnerability group, the results are less clear, since the approaches used to measure intensity of vulnerability provide opposite results both at the national level and by area of residence (Table 3.A). However, it is interesting to emphasize that the decline in severe vulnerability registered at the national level conceals an increase in the percentage of the severe vulnerability in urban areas which may suggest that some moderately vulnerable people have slipped into severe vulnerability. As a result, in Tunisia, vulnerability appears to be shifting more towards severe vulnerability as the contribution of severe vulnerability to overall vulnerability (V_{0P}^{MP}/V_{0}^{MP}) increased from 32.4% in 2012 to 43.4% in 2018.

We now take a closer look at the dimensional composition of vulnerability and its two main components. Our aim is to examine whether severe vulnerability differs from moderate vulnerability in terms of its dimensional composition. To accomplish this, we computed deprivation rates in each indicator among all the vulnerable, as well as separately for the severely and moderately vulnerable subgroups.

Figure 3. Deprivation ratio by dimension among the severely and moderately vulnerable



Source: Author's calculation based on UNICEF-MICS data.

For ease the presentation, Figure 2 displays the deprivation rates among these different groups of vulnerable for the last year of the survey in Algeria and Tunisia.

For comparison purpose, we also report the deprivation rates among the multidimensionally poor. Figure 3 complements Figure 2 by presenting the evolution over time of the ratios of deprivation rates among the severely vulnerable to the deprivation rate among the moderately vulnerable for each indicator and country.

Figure 2 shows that in Algeria, the indicators with the highest risk of deprivation among the vulnerable population are assets, overcrowding, and floor materials, followed by school attendance and years of education. The lowest risks are found in sanitation, access to water and the three health indicators (nutrition, mortality and early pregnancy). However, some deprivations clearly distinguish severe from moderate vulnerability. Figures 2 and 3 show that the most pronounced differences between severe and moderate vulnerability occur in early pregnancy, mortality, nutrition, access to water and school attendance. These dimensions correspond to structural poverty. In contrast, deprivations in the remaining indicators are more similar between the two groups of vulnerable individuals. Figure 3 also shows that deprivation ratios for mortality, nutrition and schooling increased between 2013 and 2019. Results by area of residence (Figure 1.A.) indicate that early pregnancy, mortality, access to water and nutrition differentiate the two types of vulnerable individuals, particularly in urban areas. This is evident from the much higher deprivation ratios between severe and moderate vulnerability in urban than in rural areas (Figure 2.A).

For Tunisia, the indicators with the highest risk of deprivation among the vulnerable are assets, years of education, overcrowding and floor materials. Similar to Algeria, the indicators that most distinguish severe from moderate vulnerability are mortality, early pregnancy, sanitation, nutrition followed by school attendance. Figure 3 shows that differences between the two vulnerability types widened between 2012 and 2018, especially for mortality, sanitation and school attendance. It is worth noting that deprivations in sanitation and nutrition have also increased among the severely vulnerable, although these data are not reported. Finally, Figure 2.A shows that the ratios distinguishing severe from moderate vulnerability are significantly higher in urban than in rural areas and these gaps have widened over time.

This analysis helps identify the indicators and dimensions that require specific attention in designing and implementing social policies.

3.5. Overlap between vulnerability and multidimensional poverty

As shown in Table 4, the percentage of individuals vulnerable to multidimensional poverty is about 1.5 times higher in Algeria in 2019 and 1.8 times higher in Tunisia in 2018 than the percentage of the observed multidimensional poor. It is therefore important to examine the overlap between different forms of vulnerability and poverty to identify those at risk of remaining poor or falling into poverty, as well as those likely to escape poverty. To do so, we divided the vulnerable population into four distinct groups based on severe vulnerability, moderate vulnerability, and observed multidimensional poverty. Following Chaudhuri et al. (2002) and Feeny and McDonald (2016), this classification allows us to distinguish the chronic poor, the transitory (frequently poor), the highly (severely) vulnerable non-poor (vulnerable to chronic poverty), and the relative (moderately) vulnerable non-poor. Table 6 presents this cross-tabulation for last survey year in Algeria and Tunisia.

Table 6. Vulnerability to poverty and observed multidimensional poverty

	Algeria 2019			
	Observed multidimensional poverty			
	Current poor 11.25%		Current non-poor 88.75%	
Estimated vulnerability	Total vulnerability 17.64%			
		Chronic poor 4.33%	Vulnerability to chronic poverty 2.21%	Severe vulnerability 6.54%
		Frequently poor 2.97%	Vulnerability to frequent poverty 8.13%	Moderate vulnerability 11.10%
	Not Vulnerable 82.36%	Infrequently poor 3.95%	Not vulnerable and not poor 78.41%	
	Tunisia 2018			
	Observed multidimensional poverty			
	Current poor 11.23%		Current non-poor 88.77%	
Estimated vulnerability	Total vulnerability 20.96%			
		Chronic poor 5.43%	Vulnerability to chronic poverty 3.66%	Severe vulnerability 9.09%
		Frequently poor 2.36%	Vulnerability to frequent poverty 9.52%	Moderate vulnerability 11.88%
	Not Vulnerable 79.04%	Infrequently poor 3.44%	Not vulnerable and not poor 75.60%	

Shaded area is vulnerability.
Source: Author's calculation adapted from Chaudhuri et al. (2002) and Tesliuc and Lindert (2004).

Table 7 complements Table 6 by showing vulnerability incidence by poverty status for each country and year. Results by area of residence are reported in Figure 3.A. in the Appendix. As shown in Table 6, in the last survey year, 17.64% of Algerians and 20.96% of Tunisians are vulnerable, among them, 62.9% and 56.6% respectively, are vulnerable due to transitory factors. Chronic poverty is more prevalent in Tunisia (43.4%) than in Algeria (37.1%). Among the currently poor, 38.5% in Algeria and 48.3% in Tunisia remain chronically poor with a high probability of experiencing multidimensional poverty in the future. In addition, 26.4% in Algeria and 21% in Tunisia face frequent poverty due to volatility in their expected well-being (moderate vulnerability). Finally, among the currently poor, 26.4% in Algeria and 21% in Tunisia are infrequently poor, suggesting that they are likely to escape poverty (Table 7). Among the non-poor (around 88% in both countries), only 2.5% in Algeria and 4.1% in Tunisia are vulnerable to chronic poverty. Figure 4 also illustrates the evolution of vulnerability by poverty status over time, revealing clearly distinct trajectories in the two countries. In Algeria, the proportion of severely vulnerable individuals among the poor decreased significantly from 49% in 2013 to 38.5% in 2019, while moderate vulnerability slightly increased from 25.1% to 26.4%. This trend is particularly evident in rural areas (Figure 3.A). whereas both severe and moderate vulnerability decreased in urban areas. Among the non-poor, both forms of vulnerability declined at a similar rate, though trajectories differ slightly by area: in rural areas, the decline poor was driven by a larger decrease in severe vulnerability, while in urban areas the opposite occurred (Figure 3.A.).

In Tunisia, moderate vulnerability among the poor decreased (from 38.5% in 2012 to 21% in 2018), while severe vulnerability increased (from 39.4% in 2012 to 48.3% in 2018), a pattern observed in both urban and rural areas (Figure 3.A).

Among the non-poor, the overall decline is mainly due to moderate vulnerability (from 17.8% to 10.7%), while severe vulnerability remained largely unchanged, despite a slight increase in urban areas (Figure 3.A).

Table 7. Incidence of vulnerability by poverty status in Algeria and Tunisia

		Severe	Moderate	Non-vulnerable
Tunisia 2018	Vulnerable	9,1	11,9	79,0
	Non-poor	4,1	10,7	85,2
	Poor	48,3	21,0	30,6
Tunisia 2012	Vulnerable	10,3	21,4	68,3
	Non-poor	4,1	17,8	78,2
	Poor	39,4	38,5	22,1
Algeria 2019	Vulnerable	6,5	11,1	82,4
	Non-poor	2,5	9,2	88,3
	Poor	38,5	26,4	35,1
Algeria 2013	Vulnerable	18,4	26,7	54,9
	Non-poor	7,7	27,3	65,0
	Poor	49,0	25,1	26,0

Source: Author's calculation based on UNICEF-MICS data.

4. CONCLUSION

Our objective was to assess the levels and trends of vulnerability to multidimensional poverty in Algeria and Tunisia. Unlike previous studies using the Chaudhuri et al. (2002) approach, we followed Gallardo's (2022) method, which models the joint probability of being poor and deprived in each dimension using multidimensional Bayesian network classifiers. We used the VMR (vulnerability by mean risk) approach with the standard downside semi-deviation as the risk parameter. To our knowledge, this study is currently the only application that outside Gallardo's study (2022) for Latin America. Analogous to the FGT-alpha poverty measures, we provided measures of vulnerability to multidimensional poverty for $\gamma = 0, 1$ and 2, and decomposed vulnerability into severe and moderate vulnerability. Four key findings emerged.

Firstly, in both Algeria and Tunisia, vulnerability headcount ratios are significantly higher than poverty headcount ratios indicating that relying solely on poverty estimates provides an incomplete picture. Despite similar levels of multidimensional poverty, vulnerability levels are higher in Tunisia than in Algeria. Moreover, progress in poverty reduction appears more fragile in Tunisia than in Algeria. Similar patterns are also observed for intensity and inequality among the poor.

Second, moderate vulnerability outweighs severe vulnerability in both countries. Over time, Algeria, there shows a shift towards moderate vulnerability as the contribution of severe vulnerability to overall vulnerability declined between 2013 and 2019. In contrast, in Tunisia, moderate vulnerability decreased, particularly in rural areas, while severe vulnerability increased in urban areas, suggesting that some moderately vulnerable individuals transitioned to severe vulnerability. As a result, Tunisia's vulnerability structure appears to have shifted toward severe vulnerability between 2012 to and 2018.

Third, the dimensional decomposition of vulnerability allowed us to identify the indicators where differences between severe and moderate vulnerability among the vulnerable population are most pronounced. In Algeria, these include early pregnancy, mortality, nutrition, access to water and school attendance. In Tunisia, they are mortality, early pregnancy, sanitation, nutrition followed by school attendance. These dimensions correspond to structural poverty, while deprivations in other indicators are more similar between the two groups of vulnerable individuals. This differentiation is crucial for informing the design and implementation of targeted social policies. It is particularly concerning that vulnerability risks have increased in nutrition in both countries, early pregnancy in Algeria and sanitation in Tunisia.

Fourth, the overlap analysis between vulnerability and poverty revealed important patterns. Chronic poverty among the vulnerable is more prevalent in Tunisia than in Algeria. Among the currently poor, 38.5% in Algeria and 48.3% in Tunisia remain chronically poverty, while 26.4% in Algeria and 21% in Tunisia are infrequently poor, suggesting that they are likely to escape poverty. These findings highlight divergent vulnerability trajectories in the two countries. In Algeria, severe vulnerability among the poor decreased between 2013 and 2019, while moderate vulnerability slightly increased, particularly in rural areas; in Tunisia moderate vulnerability among the poor fell as severe vulnerability rose between 2012 and 2019.

These results underscore notable differences in the nature of vulnerability in these two countries, likely reflecting the distinct social policies implemented after the “Arab Spring”.

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APPENDIX

A.1. Detailed presentation of Bayesian network classifier

A Bayesian network is a probabilistic graphical model that represents a set of random variables with their conditional dependencies using a directed acyclic graph (DAG). The nodes in the graph (DAG) represent random variables, while the edges indicate the conditional dependencies between them. For instance, an arc from Y to X_j indicates that a value taken by X_j depends on the value taken by Y. Y is considered the parent of X_j, and X_j is referred to as the child of Y. This terminology can be extended to include the descendants of a node X_j, which are the nodes that can be reached by following the arcs from X_j. In addition, the structure of the network encodes that each node is conditionally independent of its non-descendants given its parents. This condition is important for the factorization of the joint probability distribution over the entire set of random variables[8].

More formally, a BN is a pair $B = \{G, \Theta\}$ where G is a directed acyclic graph (DAG) whose nodes are the random variables and Θ a set of parameters that quantifies the dependencies between the variables within G, Θ contains the conditional probability distributions. It is formed by a parameter $\theta_{xj|pa_{(x_j)}} = P\left(x_j|pa_{(x_j)}\right)$ for each possible values x_j of X_j, given each combination of the direct parent variables of X_j denoted by $(pa_{(x_j)})$. The network then represents the following joint probability distribution:

$$P(X_1, \dots, X_q) = \prod_{i=1}^{q} P\left(x_j|pa_{(x_j)}\right)$$

In a Bayesian network classifier, variables are divided into class variables Y and feature variables $X = (X_1, \dots, X_q)$ of binary or categorical variables.

[8] This property is used to reduce the number of parameters required to characterize the joint probability distribution (JPD).

The class variable Y has no parent and each attribute X_j has the class variable(s) as parents. BN computes the joint probability distribution as:

$$P(Y, X_1, \dots, X_q) = P(Y) \prod_{i=1}^{q} P(X_j|Y)$$

Classification consists in learning the posterior conditional distribution of Y given the features $(X_{1,\dots,}, X_q)$. For an instance of the feature variables X, the most probable assignment of Y, is obtained by maximum posterior (MAP) estimation:

$$argmax_y P(Y = y|x_1, \dots, x_q)$$

where the corresponding posterior conditional probabilities $P(Y|X)$ is computed using Bayes' rule as $P(Y|X) = P(Y, X_1, \dots, X_q)/ P(X)$.

Table 1.A. Discretisation of feature household variables

Households characteristics		Algeria 2013	Algeria 2019	Tunisia 2012	Tunisia 2018
Household head gender	Woman	0.11	0.11	0.15	0.16
	Man	0.89	0.89	0.85	0.84
Household head age	less than 35	0.98	0.08	0.09	0.08
	36-45	0.34	0.34	0.33	0.33
	46-55	0.35	0.35	0.35	0.36
	56-65	0.24	0.23	0.23	0.23
Household head education	no	0.32	0.28	0.27	0.21
	primary	0.46	0.47	0.37	0.34
	secondary and higher	0.22	0.25	0.36	0.45
Household size	1 person	0.08	0.11	0.21	0.23
	2 persons	0.12	0.13	0.16	0.18
	3 persons	0.18	0.19	0.22	0.25
	4 persons	0.19	0.23	0.21	0.20
	5 persons	0.16	0.16	0.11	0.09
	6 persons	0.27	0.09	0.05	0.04
	7 persons or more		0.09	0.04	0.02
Area of residence		0.65	0.64	0.68	0.71
		0.35	0.36	0.32	0.29
Region	1	0.36	0.33	0.25	0.26
	2	0.15	0.15	0.15	0.14
	3	0.15	0.17	0.11	0.12
	4	0.06	0.07	0.23	0.23
	5	0.14	0.14	0.05	0.12
	6	0.05	0.05	0.04	0.08
	7	0.09	0.09	0.03	0.05
	8			0.09	
	9			0.05	

Notes: Regions in Algeria are: Nord Centre. Nord Est. Nord Ouest. Hauts Plateaux Centre. Hauts Plateaux Est. Hauts Plateaux Ouest. Sud. Regions in Tunisia 2012: District Tunis. Nord Est. Nord Ouest. Centre Est . Kasserine. Kairouan. Sidi Bouzid. Sud Est. Sud Ouest. Tunisia 2018: District Tunis. Nord Est. Nord Ouest. Centre Est . Centre Ouest. Sud Est. Sud Ouest. Values computed using the household as the unit of analysis.
Source: Author's calculation based on UNICEF-MICS data.

Table 2.A. Multidimensional poverty and vulnerability by area of residence

		Observed multidimensional poverty				Vulnerability based on dimensional vulnerability			
		H	M_0^1	A	M_0^2	VH	VM_0^1	VMA	VM_0^2
Algeria									
2013	**Urban**	0.189	0.083	0.438	0.038	0.352	0.163	0.462	0.081
		(0.006)	*(0.003)*	*(0.003)*	*(0.001)*	*(0.007)*	*(0.003)*	*(0.003)*	*(0.002)*
	Rural	0.381	0.185	0.485	0.094	0.623	0.336	0.539	0.192
		(0.013)	*(0.007)*	*(0.003)*	*(0.004)*	*(0.010)*	*(0.006)*	*(0.003)*	*(0.004)*
2019	**Urban**	0.067	0.028	0.424	0.012	0.080	0.031	0.386	0.013
		(0.003)	*(0.001)*	*(0.003)*	*(0.001)*	*(0.003)*	*(0.001)*	*(0.004)*	*(0.001)*
	Rural	0.191	0.085	0.445	0.039	0.343	0.151	0.441	0.073
		(0.010)	*(0.005)*	*(0.004)*	*(0.002)*	*(0.010)*	*(0.005)*	*(0.003)*	*(0.002)*
ARC Urban		-0.159	-0.163	-0.005	-0.169	-0.220	-0.243	-0.030	-0.262
ARC Rural		-0.109	-0.122	-0.014	-0.135	-0.095	-0.124	-0.033	-0.150
Tunisia									
2012	**Urban**	0.076	0.032	0.428	0.014	0.123	0.049	0.404	0.022
		(0.006)	*(0.003)*	*(0.005)*	*(0.001*	*(0.006)*	*(0.003)*	*(0.006)*	*(0.001)*
	Rural	0.367	0.169	0.461	0.082	0.690	0.309	0.447	0.153
		(0.018)	*(0.009)*	*(0.005)*	*(0.005)*	*(0.012)*	*(0.006)*	*(0.004)*	*(0.004)*
2018	**Urban**	0.057	0.023	0.409	0.010	0.095	0.033	0.348	0.013
		(0.004)	*(0.002)*	*(0.004)*	*(0.001)*	*(0.005)*	*(0.002)*	*(0.005)*	*(0.001)*
	Rural	0.232	0.103	0.445	0.048	0.457	0.196	0.429	0.092
		(0.014)	*(0.006)*	*(0.005)*	*(0.003)*	*(0.014)*	*(0.007)*	*(0.004)*	*(0.004)*
ARC Urban		-0.047	-0.054	-0.008	-0.063	-0.041	-0.064	-0.025	-0.088
ARC Rural		-0.073	-0.079	-0.006	-0.085	-0.067	-0.073	-0.007	-0.081

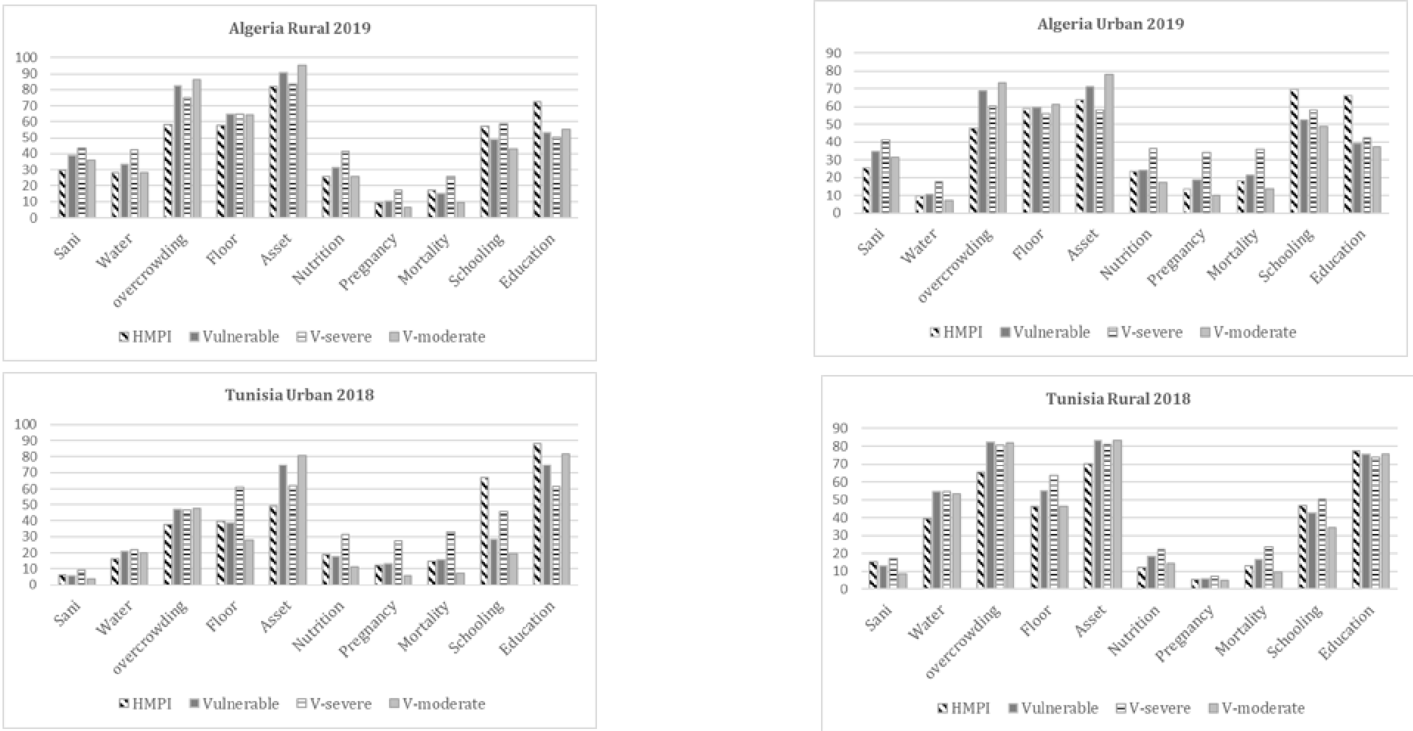
Note: ARC is the average annualized relative change. Standard errors are reported between brackets. ARC were statistically significant at α=0.01. Source: Author's calculation based on UNICEF-MICS data.

Table 3.A. Decomposition of vulnerability into severe and moderate levels by area of residence

			H	V_0^{MP}	V_{0P}^{MP}	V_{0R}^{MP}	V_{A01P}^{MP}	V_{A01R}^{MP}	V_{A1P}^{MP}	V_{A1R}^{MP}	V_{0P}^{MP}/V_0^{MP}
	2013	**Urban**	0.189	0.352	0.103	0.250	0.493	0.449	0.261	0.212	0.291
			(0.006)	*(0.007)*	*(0.004)*	*(0.005)*	*(0.004)*	*(0.003)*	*(0.007)*	*(0.003)*	
Algeria		**Rural**	0.381	0.623	0.325	0.298	0.560	0.515	0.471	0.233	0.522
			(0.013)	*(0.010)*	*(0.009)*	*(0.007)*	*(0.004)*	*(0.004)*	*(0.010)*	*(0.006)*	
	2019	**Urban**	0.067	0.080	0.028	0.052	0.441	0.356	0.330	0.175	0.351
			(0.003)	*(0.003)*	*(0.002)*	*(0.002)*	*(0.007)*	*(0.005)*	*(0.014)*	*(0.007)*	
		Rural	0.191	0.343	0.130	0.213	0.481	0.417	0.356	0.166	0.379
			(0.010)	*(0.010)*	*(0.008)*	*(0.002)*	*(0.006)*	*(0.004)*	*(0.015)*	*(0.005)*	
ARC		**Urban**	-0.159	-0.220	-0.195	-0.231	-0.019	-0.038	0.040	-0.032	
ARC		**Rural**	-0.109	-0.095	-0.142	-0.054	-0.025	-0.035	-0.046	-0.054	
	2012	**Urban**	0.076	0.123	0.028	0.095	0.495	0.377	0.206	0.237	0.227
			(0.006)	*(0.006)*	*(0.003*	*(0.005)*	*(0.011)*	*(0.007)*	*(0.019)*	*(0.013)*	
Tunisia		**Rural**	0.367	0.690	0.247	0.443	0.535	0.399	0.330	0.239	0.357
			(0.018)	*(0.012*	*(0.012*	*(0.013*	*(0.007*	*(0.004*	*(0.015*	*(0.007*	
	2018	**Urban**	0.057	0.095	0.031	0.064	0.415	0.315	0.303	0.151	0.327
			(0.004)	*(0.005)*	*(0.003)*	*(0.003)*	*(0.008)*	*(0.005)*	*(0.021)*	*(0.007)*	
		Rural	0.232	0.457	0.220	0.237	0.463	0.398	0.441	0.130	0.482
			(0.014)	*(0.014)*	*(0.011)*	*(0.008)*	*(0.006)*	*(0.007)*	*(0.014)*	*(0.005)*	
ARC		**Urban**	-0.047	-0.041	0.019	-0.063	-0.029	-0.029	0.067	-0.073	
ARC		**Rural**	-0.073	-0.067	-0.019	-0.099	-0.024	0.000	0.050	-0.097	

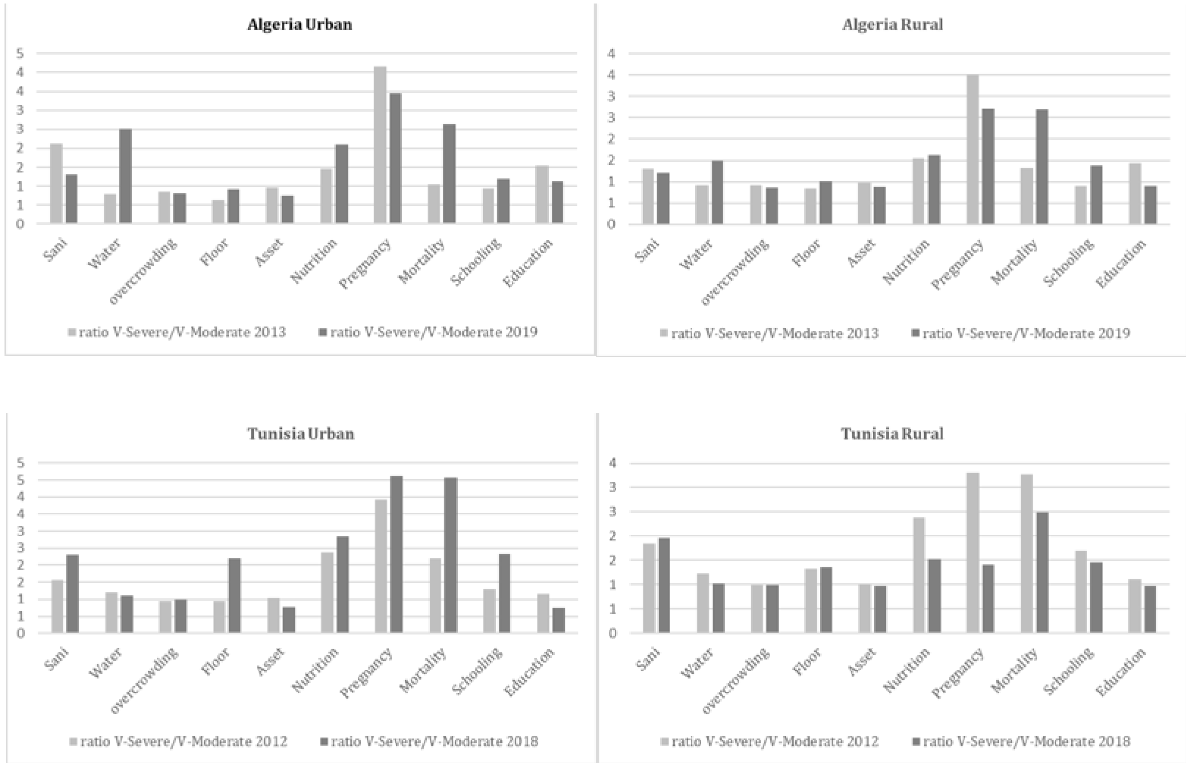
Note: ARC is the average annualized relative change. Standard errors are reported between brackets. ARC are statistically significant at α=0.01. Values of ARC for V_{A01P}^{MP} and V_{A01R}^{MP} are easier to interpret by considering the absolute variation which gives outcomes in terms of share of weighted dimensions. Source: Author's calculation based on UNICEF-MICS data.

Figure 1.A. Deprivation rates by dimension among the vulnerable by areas of residence



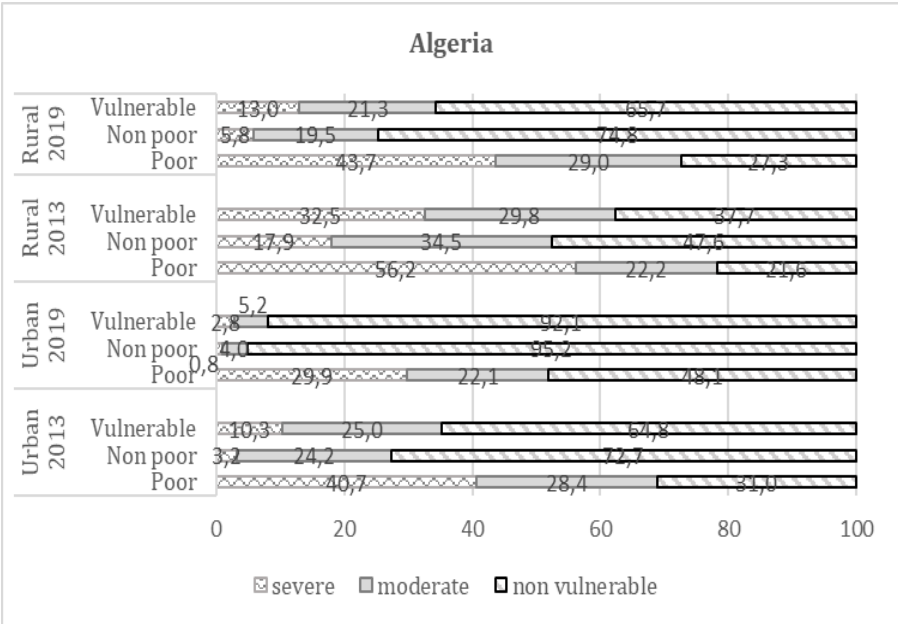
Note: V-Severe and V-moderate refer to the severe vulnerable and to the moderate vulnerable groups of people respectively. HMPI is the headcount ratio of multidimensional poverty. Source: Author's calculation based on UNICEF-MICS data.

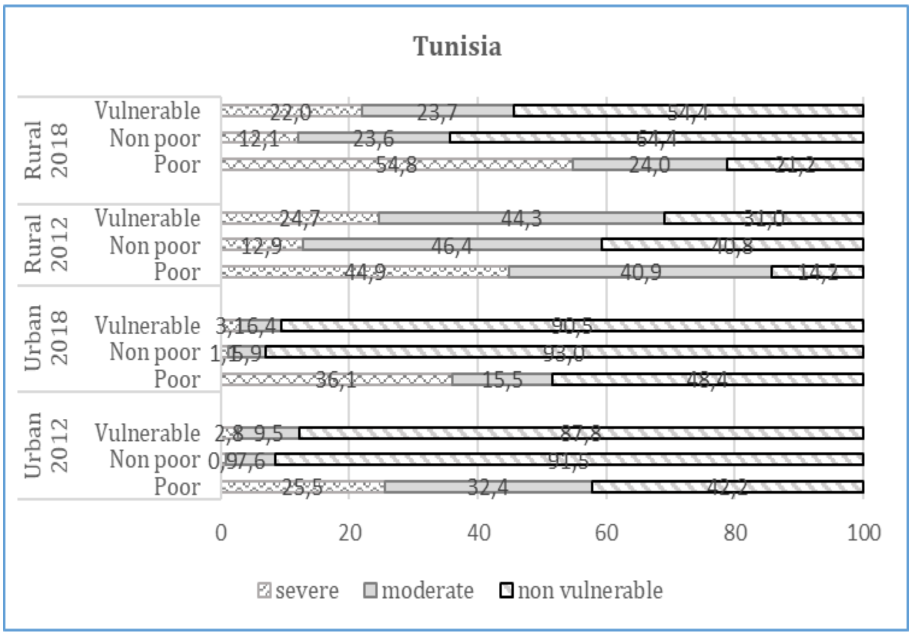
Figure 2.A. Deprivation ratio by dimension among the severe and moderate vulnerable by areas of residence



Source: Author's calculation based on UNICEF-MICS data.

Figure 3.A. Incidence of vulnerability by poverty status in Algeria and Tunisia by area of residence





Source: Author's based on UNICEF-MICS data.

Vulnérabilité à la pauvreté multidimensionnelle en Algérie et en Tunisie : une approche par comptage des privations avec des classificateurs bayésiens

Résumé - Cet article évalue la vulnérabilité à la pauvreté multidimensionnelle en Algérie (2013 et 2019) et en Tunisie (2012 et 2018). En utilisant le cadre de la « vulnerability-by-mean-risk » de Gallardo (2022) combiné aux mesures M-gamma d'Alkire et Foster (2019), nous modélisons la probabilité conjointe de pauvreté et de privations à l'aide de classificateurs bayésiens multidimensionnels. L'étude étend à la région MENA l'approche de Gallardo en examinant la vulnérabilité par dimension parmi les individus vulnérables ainsi que son recoupement avec la pauvreté. Les résultats montrent que la Tunisie présente une vulnérabilité plus élevée que l'Algérie, malgré des niveaux similaires de pauvreté multidimensionnelle. La vulnérabilité a globalement diminué, la vulnérabilité modérée étant dominante dans les deux pays, mais les tendances divergent : l'Algérie a évolué vers une vulnérabilité modérée (2013-2019), tandis que la Tunisie s'est orientée vers une vulnérabilité sévère (2012-2018). La pauvreté chronique est plus répandue en Tunisie qu'en Algérie, et les niveaux de santé et d'éducation jouent un rôle clé pour distinguer la vulnérabilité sévère de la vulnérabilité modérée dans les deux pays. Ces résultats mettent ainsi en évidence des trajectoires contrastées des composantes de la vulnérabilité dans les deux contextes nationaux.

Mots-clés
Vulnérabilité
Pauvreté multidimensionnelle
Classificateur bayésien multidimensionnel
Déviation négative moyenne
Algérie
Tunisie